Three-dimensional analysis of windows soundproofing for countries with tropical climates

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A model for manufacturing windows which are capable of ventilating, reducing traffic noise and so on for the developing tropical countries with tropical climates is presented. These windows combine two basic components which are ventilation unit and lighting unit. The former also serves as an import function in reducing noise. Due to the fact that the ventilation unit must have a large volume to attenuate low frequency noise, many resonance of higher-order mode wave will be generated inside the unit. In this work, a method to predict the insertion-loss of rectangular ventilation unit with input and output openings at various positions is proposed by solving the wave equation, considering the resonance frequencies of higher-order mode. The results of the analysis have been confirmed by experiments.

Key words: higher order mode, insertion loss, windows, wave equation.

1. INTRODUCTION

Sealing up type's doors and windows are widely used in the current houses to intercept an inside outside. Needless to say, some equipment such as air conditioners are necessary to keep a comfortable temperature indoor while such doors and windows are closed and the using duration of such equipments is often limited because of the power consumption cost and health.

Being popular in countries with tropical climates, casement windows are consisted of two wooden frames that can be opened and closed at various angles. The windows are typically opened during the day for air, naturally ventilating the room, and closed at night or when it rains. Even when closed, room ventilation is still achieved because the windows are constructed with ventilating slits. However, the annual increase in traffic noise and number of motorcycles and automobiles in developing tropical countries have rendered these windows to be useless because the ventilating slits on these windows serve as a direct pathway for traffic noise to enter the home. Consequently, windows types which are most suitable for such developing tropical countries must be able to ventilate, regulate light, and protect against the cold. In addition, they must have the ability to reduce noise and pollution from motor vehicles. Not only for the tropical climate countries, this kind of windows is thought to be possibly applied to countries around the world in order to prevent global warming that we have now facing, namely, refrain from the use of energy that destroys the natural environment.

In this paper, we present a conceptual model for manufacturing windows to achieve the above goals. These windows combine two basic components which are ventilation unit and lighting unit. The former also serves as an import function in reducing noise. Due to the fact that the ventilation unit must have a large volume to attenuate low-frequency noise, many resonance of higher-order mode wave will be generated inside the unit. Consequently, it is necessary to take into consideration the selection of size and placement of input and output openings in such a way that would minimize the effects of higher-order mode in order to have a great soundproofing effect. In this article, a three dimensional analysis method to predict the insertion loss of rectangular ventilation unit is proposed.

2. WINDOW'S DESIGN AND ANALYSIS

As shown in Fig. 1, the proposed window combines two basic components: ventilation and lighting. The lighting unit can be constructed using one or, for increased soundproofing ability, two glass layers which are mounted at an incline between two ventilation components with input and output openings. This particular design is able to prevent rain from entering the room.

In addition, we could add an accessory part that would close all input or output openings to prevent cold air or fumes from motor vehicles or insects from entering.

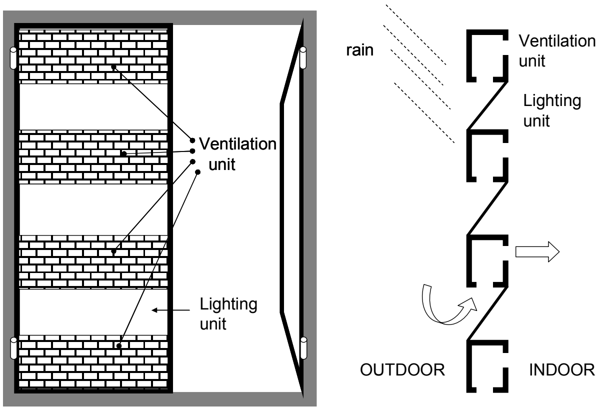

Fig.1 Design of SPCW

Let we find the sound pressure on the input and output of the ventilation unit as shown in Fig.2 The input and output have a sectional area of $S_i = 2\alpha_i \times 2\beta_i$ and $S_0 = 2\alpha_0 \times 2\beta_0$ which its center is located at point $I\,(x_i, y_i, 0)$ and $O\,(x_0, b, z_0)$, respectively.

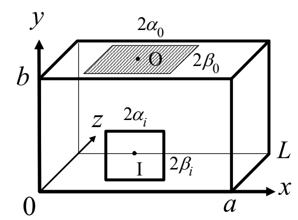

Fig. 2 model of ventilation unit

The complete of the wave-equation in terms of the velocity potential ϕ when expressed in rectangular coodinates is given by

$$\phi = \left(Ae^{\mu z} + Be^{-\mu z} \right)\left(C\sin\alpha x + D\cos\alpha x \right)$$

$$\left(E\sin\sqrt{s^2-\alpha^2}\,y + F\cos\sqrt{s^2-\alpha^2}\,y \right) \quad (1)$$

where A, B, C, D, E and F are arbitrary constants determinable from the boundary conditions, α, *s* and μ are constants.

Let $V_x = -\partial\phi/\partial x$, $V_y = -\partial\phi/\partial y$ and $V_z = -\partial\phi/\partial z$ are the velocity component in the x, y and z directions, respectively. Assuming the walls of the cavity to be perfectly rigid and the loss at the wall can be neglected, the boundary conditions are

[1] at $x = 0$, $V_x = 0$ (2)

[2] at $x = a$, $V_x = 0$ (3)

[3] at $y = 0$, $V_y = 0$ (4)

[4] at $y = b$, $V_y = 0$ (5)

[5] at $z = 0$, $V_z = V_i\,F_i(x, y)$ (6)

[6] at $z = L$, $V_z = 0$ (7)

where V_i is the driving velocity at the input, $F_i(x, y)$ is the function which has a value of 1 at the input section area and 0 at the other area, namely $(x_i - \alpha_i \le x \le x_i + \alpha_i$, $y_i - \beta_i \le y \le y_i + \beta_i)$

After the computation based on Eq.(2) until Eq.(7) is performed, the sound pressure on the side where the output located can derived as

$$P_0 = jkZ_wU_iS_w \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{\cosh\left(\mu_{m,n}(z-L)\right)}{\mu_{m,n}\sinh\left(\mu_{m,n}L\right)} D_{m,n} \cos\left(\frac{m\pi x}{a}\right)\cos\left(n\pi\right) \quad (8)$$

where

$$\mu_{m,n} = \sqrt{\left(m\pi/a\right)^2 + \left(n\pi/b\right)^2 - k^2} \quad (9)$$

$$D_{m,n} = \frac{16}{m\,n\,\pi^2 S_i}\cos\left(\frac{m\pi x_i}{a}\right)\cos\left(\frac{n\pi y_i}{b}\right) \times \sin\left(\frac{m\pi\alpha_i}{a}\right)\sin\left(\frac{n\pi\beta_i}{b}\right) \quad (10)$$

$U_i = V_i S_i$ is the volume velocity at input.

k and Z_w are the wave number and cavity impedance, respectively. Therefore, the average output sound pressure becomes

$$\overline{P}_0 = \frac{1}{S_0}\int_{x_0-\alpha_0}^{x_0+\alpha_0}\int_{z_0-\beta_0}^{z_0+\beta_0} P\,(x, b, z)\,dz\,dy$$

$$= jkZ_wU_i \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{\overline{R}_{m,n}}{\mu_{m,n}\sinh\left(\mu_{m,n}L\right)} \quad (11)$$

where

$$\overline{R}_{m,n} = \frac{S_w}{S_0} D_{m,n}\cos\left(n\pi\right) \int_{x_0-\alpha_0}^{x_0+\alpha_0}\int_{z_0-\beta_0}^{z_0+\beta_0} \cosh\left(\mu_{m,n}(z-L)\right)\cos\left(\frac{m\pi x}{a}\right)dz\,dx \quad (12)$$

Moreover, expanding the above equation corresponding to variation of *m* and *n*, we have

$$\overline{P}_0 = jZ_wU_i \left\{ -\frac{\overline{R}_{0,0}}{k\sin\left(kL\right)} + \sum_{m=1}^{\infty} \frac{\overline{R}_{m,0}}{\mu_{m,0}\sinh\left(\mu_{m,0}L\right)} + \sum_{n=1}^{\infty} \frac{\overline{R}_{0,n}}{\mu_{0,n}\sinh\left(\mu_{0,n}L\right)} + \sum \frac{\overline{R}_{m,n}}{\mu_{m,n}\sinh\left(\mu_{m,n}L\right)} \right\} \quad (13)$$

where

$$\overline{R}_{0,0} = -\frac{1}{S_0}\int_{x_0-\alpha_0}^{x_0+\alpha_0}\int_{z_0-\beta_0}^{z_0+\beta_0}\cos\left(k(z-L)\right)dz\,dx$$

$$= -\frac{1}{S_0}\frac{2\alpha_0}{k}\{\ \sin(k(\beta_0+L-z_0))+\sin(k(\beta_0-L+z_0))\ \}\ (14)$$

Similarly, the average sound pressure on the input is given by

$$\overline{P}_i = \frac{1}{S_i}\int_{x_i-\alpha_i}^{x_i+\alpha_i}\int_{y_i-\beta_i}^{y_i+\beta_i} P\,(x,y,0)\,dx\,dy$$

$$= j\,k\,Z_w U_i \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\cosh\left(\mu_{m,n}L\right)}{\mu_{m,n}\sinh\left(\mu_{m,n}L\right)}\overline{E}_{m,n} \qquad (15)$$

where the level $\overline{E}_{m,n}$ corresponding to (*m, n*) modes is given as

$$\overline{E}_{m,n} = \frac{S_w}{S_i}D_{m,n}\int_{x_i-\alpha_i}^{x_i+\alpha_i}\int_{y_i-\beta_i}^{y_i+\beta_i}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)dx\,dy \quad (16)$$

On the other hand, insertion loss *IL* defined by [1]

$$IL = 10\ log\frac{W_r}{W_0} = 10\ log\left|\frac{U_1}{U_2}\right|^2 \qquad (17)$$

Here, W and W_0 are the radiated power at one point in space with or without the acoustic element inserted between that point and the source. The ratio of U_1/U_2 is equal to the *D* parameter of four-pole parameters as far as constant velocity source be concerned.

When the acoustic element is connected with 3 elements in series, as the sectional area of element 1 and 3 are sufficient small to compare with those of element 2 the *D* parameter of whole system can be described by the following approximated equation

$$D = (\cos kl_1)\,(C_w)\,(jZ_3\sin kl_3) \qquad (18)$$

where C_w denoted the four-pole parameters *C* of element 2. As shown in Eq. (17) and Eq. (18), in order to obtain a reliable *IL* effect, *D* parameter must be high enough. In other words, the design of element-2 to have a high enough C_w is demanded. Note that, C_w can be derive as

$$C_w = \frac{U_i}{\overline{P}_0} \qquad (19)$$

where $\overline{P}_0$ is defined by Eq.(13).

3. RESULTS

C_w is defined by Eq.(19) including the average output sound pressure $\overline{P}_0$ as shown in Eq.(13) at its denominator. In order to obtain an *IL* effectively C_w must be at great, in other words, low level of $\overline{P}_0$ is preferable. Referring to Eq.(13), $\overline{P}_0$ becomes great when its denominator sin(*kL*) and $\mu_{m,n}\sinh\left(\mu_{m,n}L\right)$ are zero, namely, at the following resonance frequencies of

$$\sin(kL) = 0 \qquad \therefore \qquad f_0 = \eta\frac{c}{2L} \quad (\eta = 1, 2, 3....) \qquad (20)$$

$$\mu_{m,n}\sinh\left(\mu_{m,n}L\right) = 0 \qquad \therefore$$

$$f_{m,n} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2+\left(\frac{\eta\pi}{L}\right)^2} \quad (\eta = 0, 1, 2, 3....) \qquad (21)$$

The f_0 represents the resonance frequencies of the plane wave and $f_{m,n}$ represents those of traverse wave. Generally, at the frequency range where the traverse waves are generated, the sound pressure $\overline{P}_0$ will increase and when the resonance frequencies of other modes co-occur, C_w will be small and the *IL* can not be expected to be as great. Generation mechanism of these frequencies can be understood according to the calculation example shown in Fig. 3(a) with (2,0) and (4,0) modes. Sound pressure level of those wave components in dB are shown in Fig. 3(b). They also have many resonance frequencies which occur corresponding to the increasing of η in Eq.(21). Therefore, it is clear that when we eliminate an arbitrary higher-order wave mode by any method, we will not only avoid many resonances generated by this mode but also obtain the low level of the entire output sound pressure.

Figure 4 shows the measured and theoretical results in case of point A (*a*/2, *b*, *L*/2) located on the top of the cavity. By locating at *z*=*L*/2, Eq.(11) becomes

$$P_A = P(a/2, b, L/2)$$

$$= j\,k\,Z_w U_i S_w\,\frac{1}{2}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{1}{\mu_{m,n}\sinh\left(\mu_{m,n}L/2\right)}D_{m,n}$$

$$\times\ \cos\left(\frac{m\pi}{2}\right)\cos\left(n\pi\right) \qquad (22)$$

Moreover, expanding the above equation with mode (0,0) we have

$$P_A = j\,k\,Z_w U_i S_w\,\frac{1}{2}\left\{\frac{1}{k\sin(kL/2)}\right.$$

$$\left.+\sum^{\bullet}_{\bullet}\frac{1}{\mu_{m,n}\sinh\left(\mu_{m,n}L/2\right)}D_{m,n}\cos\left(\frac{m\pi}{2}\right)\cos\left(n\pi\right)\right\} \quad (23)$$

As a result, the denominators of P_A become sin $(kL\ /2)$ and $\sinh\left(\mu_{m,n}L/2\right)$. Calculation examples of mode(0,0) and mode(4,0) are shown in Fig.4(b). The frequencies at which $\sin(kL/2)$ and $\sinh\left(\mu_{m,n}L/2\right)$ become zero correspond well to those of the measured results shown in Fig.

4(a). By using Eq.(16) the optimized positions of input can be determine as shown in Fig.5.

3. CONCLUSIONS

The characteristic of sound propagation in the rectangular ventilation has been presented by solving the wave equation, considering the higher-order mode effects. Based on the obtained results, the cause and mechanism of resonance frequencies of parameter C are discussed in detail. To prove the theory, experiments were carried out and excellent agreement is obtained. The formulas derived from the present study enable the account of the insertion loss of the ventilation in practical applications.

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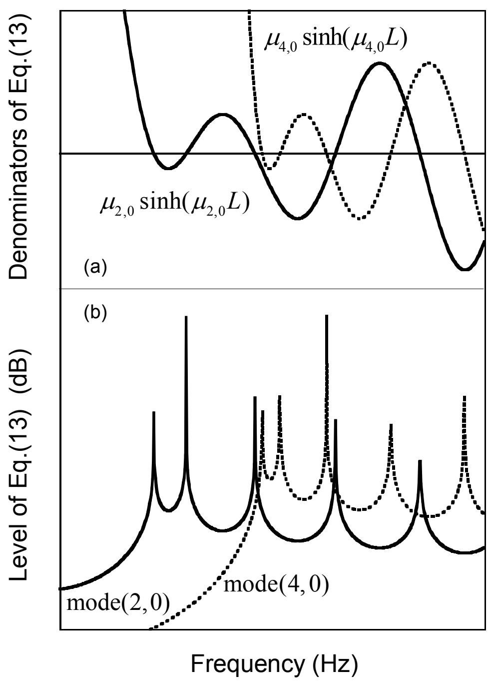

Fig.3 Physical meaning of Eq.(13)
(a) calculated example of Eq(13) with (2,0) and (4,0) mode. (b) Spectrum of (2,0) and (4,0) mode

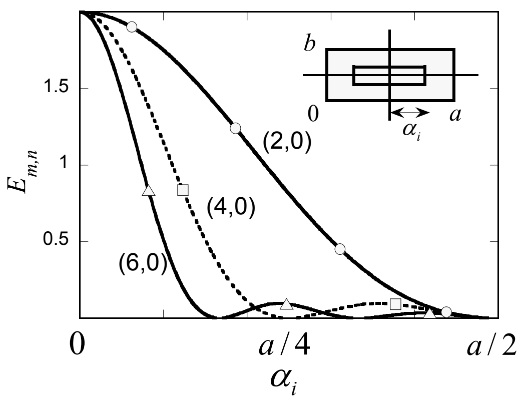

Fig.5 Sound pressure level of (m,0) mode with the variation of α_i

I *(a/2, b/2, 0)* *a=0.48m*
C *(a, b/2, L/2)* *b=0.075m*
A *(a/2, b, L/2)* *L =0.29m*

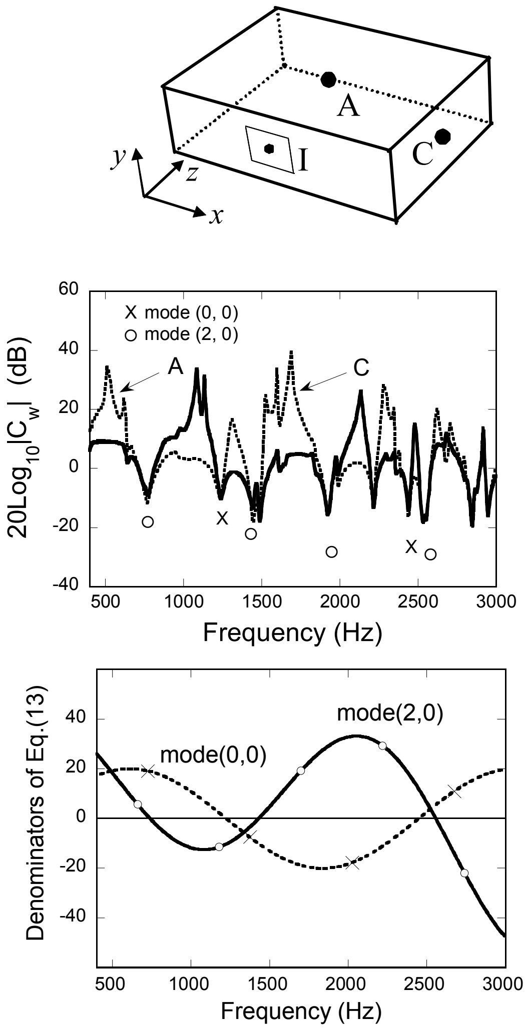

Fig.4 Resonance frequencies appear in the ventilation unit. (a) measured result (upper) (b) computed result of the denominator of Eq.(23) (lower).